III. "On the Electromotive Changes connected with the Beat of the Mammalian Heart, and of the Human Heart in particular." By AUGUSTUS D. WALLER, M.D. Communicated by Professor BURDON SANDERSON, F.R.S. Received June 12, 1888.

(Abstract.)

1. Description of experiments in which the electrical variation connected with the spontaneous beat is modified.

2. The normal ventricular variation is diphasic, and usually indicates (1) negativity of apex, (2) negativity of base.

3. Description of "irregular" variations.

4. Observations on animals with one or both leading off electrodes applied to the body at a distance from the heart.

5. Determination of the electrical variations of the heart on man.

6. The variation is diphasic, and indicates (1) negativity of apex, (2) negativity of base.

7. Distribution of cardiac potential in man and animals. "Favourable" and "unfavourable" combinations.

8. Demonstration of electrical effects by leading off from the surface of the intact body by the various extremities and natural orifices.

9. Comparison between effects observed on man with the normal and with a transposed situation of the viscera.

IV. "On the Plasticity* of Glacier and other Ice." By JAMES C. McCONNEL, M.A., Fellow of Clare College, Cambridge, and DUDLEY A. KIDD. Communicated by R. T. GLAZEBROOK, F.R.S. Received June 11, 1888.

The experiments described in the following paper were undertaken in continuation of those made by Dr. Main in the winter 1886–87, and described by him in a paper† read before the Royal Society the following summer. The investigation is by no means complete, but the results hitherto obtained seem to us sufficiently novel and important to be worthy of being put on record, while we hope to

* Dr. Main used the term "viscosity." But this has been always applied in liquids to molecular friction, and we have the authority of Sir Wm. Thomson ('Encycl. Britann.,' Art.: *Elasticity*, p. 7) for reserving it for the same property in solids also, leaving "plasticity" to denote continuous yielding under stress.

† 'Roy. Soc. Proc.,' vol. 42, p. 329.

prosecute the subject further next winter. We shall first give a general account of our results, and then describe the experiments in more full detail.

Main found that a bar of ice, which had been formed in a mould,* yielded slowly but continuously to tension, though kept at a temperature some degrees below freezing point. We began work under the impression that the rate of extension depended mainly on the temperature and tension, and that the chief difficulty lay in keeping the temperature constant. But by a happy chance our very first experiment showed us that not merely the rate, but even the very existence of the extension depended on the structure of the ice. And this is a matter which seems to have been quite disregarded by previous experimenters.†

After many, and for the most part unsuccessful, attempts to obtain a piece of perfectly clear ice, frozen in the mould used by Main, we took a bar cut from the clear ice formed on the surface of a bath of water, and froze its ends on to blocks of ice fitting the two conical collars through which the tension is applied. To avoid any question as to the ice giving way in the collars, where it is subjected to pressure as well as tension—the bar was pierced near either end by a steel needle firmly frozen in, and the measurements were taken between the projecting ends of these needles. We found to our astonishment that the stretching was almost *nil*, though the tension was decidedly greater than that usually applied by Main. There was a slight extension at first, but during the last five days the extension observed was at the mean rate of only 0·00031 mm. per hour per length of 10 cm., and this may well be attributed to the rise of temperature which took place. The rigidity cannot have been due to the cold, for during the last 24 hours the temperature was between $-1°$ and $-2°$.‡ After the experiment, the ice was examined under the polariscope, and found to be a single regular crystal showing the coloured rings and black cross very well. The optic axis was at right angles to the length of the bar. This experiment showed it was a very necessary precaution to take the measurements between needles fixed in the bar itself. For whether the bar extended or not, the movement of the index H (fig. 2), showed

* The mould produced a round bar of ice 24 cm. in length and 2·8 cm. in diameter, with a conical expansion at the lower end to fit into an iron collar C (fig. 2), through which the tension could be applied. The other end of the bar was frozen on to ice filling a similar collar B. These iron collars were faced with carefully worked brass plates, and Main determined the extension by measuring the distance between the plates with callipers.—July 6, 1888.

† See Heim, 'Handbuch der Gletscherkunde,' published by Engelhorn, Stuttgart, 1885, p. 315.

‡ We use the centigrade scale of temperature throughout.

a decided separation of the collars due to the plasticity of the conical pieces of ice therein.

We next took a bar of ice formed in the mould, applied tension and took measurements in the same way. The extension was at the rate of 0·048 mm. per hour per length of 10 cm. The crystalline structure of this ice was highly irregular. As one principal object of our experiments lay in their application to the theory of glaciers, it had now become obviously most important to test actual glacier ice. We therefore drove over to the Morteratsch glacier, which is now readily accessible from St. Moritz even in the winter, and obtained some specimens from the natural ice caves at the foot of the glacier.

We tested three pieces, which were quite sufficient to disprove the common notions, that glacier ice is only plastic under pressure, not under tension, and that regelation is an essential part of the process. They showed at the same time the extraordinary variability of the phenomenon. The first extended at a rate of from 0·013 mm. to 0·022 mm. per hour per length of 10 cm., the variations in speed being attributable to temperature. The second piece began at a rate of 0·016 mm. and gradually slowed down till it reached at the same temperature a rate of 0·0029 mm., at which point it remained tolerably constant, except for temperature variations, till a greater tension was applied. The third piece on the contrary began at the rate of 0·012 mm., increased its speed with greater tension to 0·026 mm., and stretched faster and faster with unaltered tension, till it reached the extraordinary speed of 1·88 mm. per hour per length of 10 cm. We put on a check by reducing the tension slightly, whereupon the speed fell at once to 0·35 mm. and gradually declined to 0·043 mm. The lowest temperature reached during our experiments, except with the intractable bath ice, was with this specimen. During 12 hours with a maximum temperature $-9°$ and a mean temperature probably $-10·5°$, the rate under the light tension of 1·45 kilo. per sq. cm. was 0·0065 mm.

These three pieces were composed of a number of crystals varying in thickness from two or three millimetres up to thirty or even a hundred. These crystals are the "glacier grains" (*gletscherkörner*), which play such a large part in glacier literature. Glacier ice is a sort of conglomerate of these grains, differing, however, from a conglomerate proper in that there is no matrix, the grains fitting each other perfectly. In the winter, at any rate, the ice on the sides of the glacier caves looks quite homogeneous. But, when a piece is broken off and exposed to the sun's rays, the different grains become visible to the naked eye, being separated probably by thin films of water. Though the optical structure of each grain is found under the polariscope to be perfectly uniform, the bounding surfaces are utterly irregular, and are generally curved. The optic axes too of

neighbouring grains seem to be arranged quite at random. Owing to the structure being so complex, we failed to trace any relation between the arrangement of the crystals and the rapidity of extension. It is true that the most rigid piece of the three was composed of small crystals, while the most plastic contained one very large crystal; but this was perhaps accidental. Fortunately, we were able to obtain ice of a more regular structure, which has already thrown a little light on the action at the interfaces of the crystals, and offers an attractive field to further investigation.

Some of the ice of the St. Moritz lake is built up of vertical columns,* from a centimetre downwards in diameter, and in length equal to the thickness of the clear ice, *i.e.*, a foot or more. A horizontal section, exposed to the sun for a few minutes, shows the irregular mosaic pattern of the divisions between the columns. The thickness of each column is not perfectly uniform. Sometimes indeed one thins out to a sharp point at the lower end. Each column is a single crystal, and the optic axes are generally nearly horizontal. Some experiments on freezing water in a bath, lead us to attribute this curious structure to the first layer of ice having been formed rapidly, in air, for instance, below $-6°$ C. We found that if the first layer had been formed slowly, and was therefore homogeneous with the axis vertical, a very cold night would only increase the thickness of the ice, while maintaining its regularity.

We applied tension to a bar of lake ice carefully cut parallel to the columns. It stretched indeed, but excessively slowly. During seven days it stretched at the rate of only 0·0004 mm. per hour per length of 10 cm., though at one time the temperature of the surrounding air went up above zero. The tension was 2 kilos. per sq. cm. This slight extension may well be attributed to the tension not being exactly parallel to the interfaces of the columns. This experiment corroborates our first result, that a single crystal will not stretch at right angles to its optic axis. We next cut a bar at about 45° to the length of the columns, and the difference was very manifest. During 80 hours under a tension of 2·75 kilos. per sq. cm., it extended at the rate of 0·015 mm. per hour per length of 10 cm., nearly 40 times as fast.

An icicle is an example of ice formed of very minute crystals irregularly arranged. We found that an icicle under a tension of 2·2 kilos. per sq. cm. stretched at the rate of 0·003 mm. per hour per length of 10 cm. This is very slow, especially as the temperature

* This was the case in all pieces obtained from one end of the lake, where men were cutting ice for storage purposes, whether new ice or old. In a part, however, which had frozen a few days earlier, further out from the shore, we found much larger crystals with the axes nearly vertical but not quite parallel to each other.—July 6, 1888.

was high, averaging $-1°$ C., yet it is difficult to suggest any theoretical reason for an increase in the number of interfaces producing a decrease in the plasticity.

We tried further two experiments on compression of ice, the pressure being applied to three nearly cubical pieces at once. Of three pieces of glacier ice, under a pressure of 3·2 kilos. per sq. cm., the mean rates of contraction during five days were respectively 0·035 mm., 0·056 mm., and 0·007 mm. per hour per length of 10 cm. These figures show that while the plasticity varies enormously in different specimens, the rate of distortion is of the same order of magnitude, whether the force applied be a pull or a thrust.

The other experiment was on three pieces of lake ice, applying the pressure in a direction parallel to the columns. The contraction was scarcely perceptible. Under a pressure of 3·7 kilos. per sq. cm., the mean rate of the three pieces during four days was 0·001 mm. per hour per length of 10 cm. To fix the blocks of ice in position, we found it necessary to cover their ends with paper frozen on, and the small contraction observed may well be attributed to the yielding of the films of irregular ice with which the paper was attached. This view is supported by the fact that nearly the whole of the contraction took place in the first 36 hours.

We have now shown by direct experiment that ordinary ice, consisting of an irregular aggregation of crystals, exhibits plasticity, both under pressure and under tension, at temperatures far below the freezing point—in the case of tension at any rate down to $-9°$ at least, and probably much lower—and also that a single uniform crystal will not yield continuously either to pressure or tension when applied in a direction at right angles to the optic axis. We fully intended to test a crystal under tension applied along the optic axis; but we were unsuccessful in obtaining a crystal longer in the axis than perhaps 8 cm., and when we had decided to be content with that length, a thaw put a stop to all further operations. We have, however, very little doubt that a crystal would refuse to yield either to pressure or to tension in whatever direction they were applied.

The following reasoning seems tolerably conclusive as far as it goes. We first assume the axiom that, if two systems of stresses produce each by itself no continuous yielding, superposition of the two will likewise produce no continuous yielding. This will probably be admitted when we add the proviso that, when the nature of the resultant stresses is found, their magnitude is to be reduced to the same value as that of the simple stresses which are known to be inactive. Take then a cube of ice, two of whose faces are perpendicular to the optic axis. Apply tension to one of the other pairs of faces. This according to our experiments produces no extension.

Of course we do not take into account the slight elastic yielding. Apply an equal tension to the other pair of faces which are parallel to the axis. There is still no extension by the axiom. Now it can hardly be supposed that an uniform hydrostatic pressure could produce continuous change of form. Apply then a pressure of such magnitude as to neutralise the two tensions. We have then remaining only a pressure along the optic axis, producing no continuous yielding.

In a similar way it may be shown that tension along the optic axis would produce no continuous yielding. It is true that the reasoning cannot be extended to pressures and tensions oblique to the optic axis. But if the plasticity observed had been due to the majority of crystals extending, while a certain number remained unchanged, there would surely have been numerous cracks found in every case; while as a matter of fact such cracks were only found in two cases, and then they were very slight. Hence, while we think it desirable to experiment further in the matter, we feel tolerably confident that single crystals of ice are not plastic, and we attribute the apparent plasticity of glacier ice to some action at the interfaces of the crystals. But we are not at present inclined to venture any opinion as to the nature of this action.

The variation of plasticity with the temperature is of great interest both for the theory of glaciers and for the explanation of the plasticity itself, but it is so difficult to disentangle the temperature variations proper from the much larger alterations due to structural changes, that our experiments throw very little light on this point. In the case of the glacier ice in Experiment 7 the rate seems to have become tolerably constant except for temperature changes. While at $-3{\cdot}5^\circ$ the rate was 0·0029, two days before and two days afterwards it was about 0·0020 at -5°, and a few days earlier 0·0013 at -8°. In the icicle, when the temperature variations seemed paramount, the rate at -2° was 0·0028, and at $-0{\cdot}2^\circ$ 0·0034. This is a much smaller change than we should have expected. In the case of compression the influence of temperature seems more strongly marked. In all three pieces the rate rose at -3° to about ten times its value at -5°. An increase which takes place in three pieces simultaneously can hardly be attributed to structural changes independent of the temperature.

The change in the rate of extension, produced by an alteration of the tension, was in every case altogether out of proportion to the magnitude of the latter. In the following table are collected all the instances which occurred:—

Specimen.	Change of tension.	Change of rate.
	kg. per sq. cm.	mm. per hour per 10 cm.
Glacier ice C......	2·55 to 3·85	0·0018 to 0·0110
Glacier ice D......	1·45 ,, 2·55	0·0075 ,, 0·026
,, ...	2·55 ,, 1·03	0·105 ? ,, 0·010
,, ...	1·03 ,, 2·50	0·010 ,, 0·228
,, ...	2·50 ,, 1·80	1·88 ,, 0·35

The changes of temperature in these cases were insignificant compared with the alteration of rate. The 0·105 is uncertain owing to an accident. It was certainly not less, and may have been a good deal greater.

We append a summary of some of our results arranged in tabular form. Glacier ice C was the same piece as B, cut rather shorter.

Summary. Extension Experiments.

Experiment No.	Description of specimen.	Duration.	Rate per hour in mm. per length of 10 cm.	Tension kilos. per sq. cm.	Maximum temperature.	Mean temperature.
2	Bath ice, uncorrected for temperature	$5\frac{1}{2}$ days	0·00028	4·9	−1·0°	− 4·5°
,,	,, corrected for temperature	,,	0·00000	,,	,,	,,
3	Mould ice	28 hours	0·048	3·8	0·0	− 5·0?
4	Glacier ice A, maximum rate	5 ,,	0·022	1·66	0·0	− 2·0
,,	,, minimum rate	4 ,,	0·013	,,	−1·0	− 2·5
5	Glacier ice B, maximum rate	24 ,,	0·016	2·7	−2·5	− 3·5
7	Glacier ice C, ,,	23 ,,	0·0068	2·55	−2·5	− 4·5
,,	,, minimum rate	3 days	0·0013	,,	−6·0	− 9·0
8	Glacier ice D, maximum rate	10 mins.	1·88	2·50	−2·1	− 2·1
,,	,, minimum rate	16 hours	0·0054	1·45	−6·0	−10·0
,,	,, lowest temperature	12 ,,	0·0065	,,	−9·0	−10·5
6	Icicle, maximum rate	5 ,,	0·0041	2·2	0·0	0·0
,,	,, minimum rate	8 ,,	0·0015	,,	−0·7	− 1·7
9	Lake ice, parallel to columns	7 days	0·00039	2·1	0·0	− 5·5
,,	,, greater tension	2 ,,	0·00076	2·8	−4·0	− 5·5
10	Lake ice, oblique to columns, maximum rate	6 hours	0·034	2·75	−5·6	− 5·8
,,	Lake ice, oblique to columns, minimum rate	16 ,,	0·010	,,	,,	− 6·0

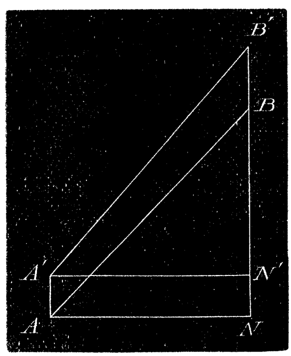
Compression Experiments.

Experiment No.	Description of specimen.	Duration.	Rate per hour in mm. per length of 10 cm.	Pressure kilos. per sq. cm.	Maximum temperature.	Mean temperature.
1	Glacier ice E	5 days	0·035	3·2	−2·8	− 6·0
,,	Glacier ice F	,,	0·056	,,	,,	,,
,,	Glacier ice G	,,	0·007	,,	,,	,,
2	Lake ice, parallel to columns, A	3 days	0·0002	3·7	−3·9	− 6·0
,,	Lake ice, parallel to columns, B	,,	0·0012	,,	,,	,,
,,	Lake ice, parallel to columns, C	,,	0·0018	,,	,,	,,

It will be interesting to make some numerical comparison between the figures we have given and the plasticity actually observed in the motion of glaciers. Perhaps the most striking proof of the existence of plasticity is the great increase of velocity from the side to the centre of a glacier. A number of measurements on this point have been collected by Heim ('Gletscherkunde,' p. 147). The most rapid increase he mentions among the glaciers of the Alps is on the Rhone glacier, on a line 2300 metres above the top of the icefall. At 100 metres from the western bank the mean yearly motion, 1874 to 1880, was 12·9 metres; at 160 metres from the bank it was 43·25 metres. This gives an increase of velocity in each metre across the glacier of 0·000058 metre per hour.

Let us consider what rate of extension this involves.

FIG. 1.



Let AB (fig. 1) be two points on a glacier moving in parallel directions, of which B is moving faster. In the small time δt (whose square we may neglect) let A move to A′ and B to B′. Draw AN, A′N′ perpendicular to B′B produced. Let AN = A′N′ = a, BN = x, B′N′ = x', AB = r, A′B′ = r', and let the velocities be v_a and v_b.

Then $\quad$ A′A $= v_a\,\delta t,\qquad$ B′B $= v_b\,\delta t,$

$$r^2 = a^2+x^2,$$

$$r'^2 = a^2+x'^2 = a^2+\{x+(v_b-v_a)\delta t\}^2$$

$$= a^2+x^2+2x\,(v_b-v_a)\,\delta t$$

$$\therefore r'^2 = r^2\left(1+\frac{2x(v_b-v_a)\delta t}{a^2+x^2}\right),$$

and
$$r' = r\left(1+\frac{x(v_b-v_a)\,\delta t}{a^2+x^2}\right);$$

so
$$\frac{r'-r}{r} = \frac{x}{a^2+x^2}\,(v_b-v_a)\ \delta t. \qquad (1.)$$

The expression $\frac{x}{a^2+x^2}$ is a maximum when $x = a$, and then we have by (1)—

$$\frac{1}{r}\frac{r'-r}{\delta t} = \frac{v_b-v_a}{2a}. \qquad (2.)$$

When δt is very small, the ratio of $r'-r$ to δt is the rate of increase of the distance between A and B. So, if we take any two points of the glacier at unit distance, the rate of increase of the distance between them will be greatest when the line joining them is at 45° to the direction of motion, and this maximum value will be equal to one half the difference of the velocities of two points situated abreast of each other and also at unit distance.

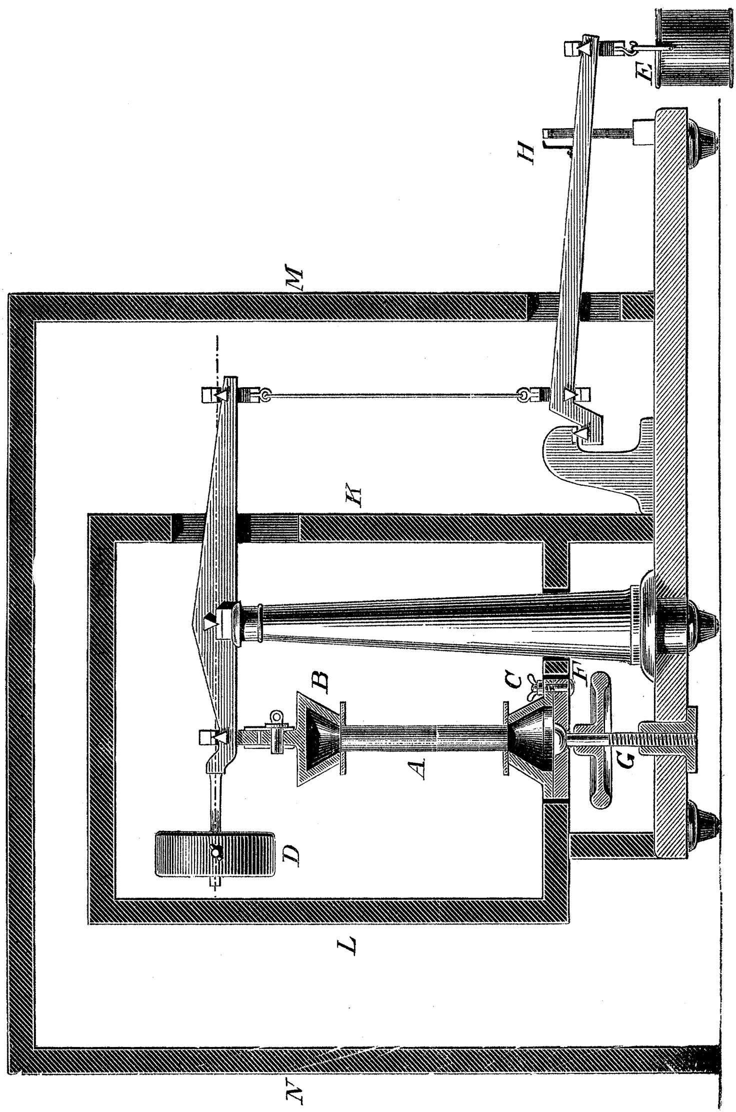
Thus the maximum rate of extension in the case we have taken on the Rhone glacier is 0·0029 mm. per hour per length of 10 cm. This, be it remembered, is the most rapid extension selected from a large number of measurements on different glaciers and at different times, and yet only one of the three specimens of glacier ice showed a rate less than this, and that was under one-third of the breaking tension. The larger the specimen, the greater average plasticity would it display; for the addition of a small piece like our second specimen, for instance, would suffice to make a long rigid bar appear very plastic. Hence the glacier itself would be far more plastic than most small specimens taken at random from its mass. It would seem, therefore, that neither the presence of crevasses nor a thawing temperature are essential conditions of the motion of a glacier. But that crevasses are found is not surprising, when we consider the rotten state of the ice during the summer and the certainty that a crack, however small, once formed will continue as long as the tension exists. We believe further that the stresses produced in a glacier by its own weight are comparable with those employed in our experiments.

Description of Apparatus.

We had two sets of apparatus in operation. The first was that employed by Dr. Main, figured and fully described in his paper. We reproduce his figure unaltered, though we made a few alterations in the surrounding boxes. As we expected at first that our chief difficulty would be keeping the temperature constant, we made special arrangements for overcoming this. To secure a large heat capacity we

introduced two tins, filled with a strong solution of salt, into the inner box, slightly altering its shape and increasing its size for this purpose. A broad shallow tin occupied the spare space at the top, and a tall tin occupied all the available space by the side of the ice between A and L (fig. 2). The space between the two boxes was filled with

FIG. 2.



wood shavings, except between K and M. Here a wooden partition P was inserted to the left of the vertical connecting rod. The space between K and P was filled with wood shavings. To allow the lever to move freely, it passed through a wooden tube loosely packed with cotton-wool. The outer space between P and M was made fairly air-tight, and the opening through which the lower lever emerged was also plugged with cotton-wool.

The capacity of the inner chamber was about 60 litres, while the two tins contained about 25 litres of solution. The inner chamber was thus jacketed on all sides with a layer, from $10\frac{1}{2}$ to 20 cm. in thickness, of which from 4 to 6 cm. was solid wood and the rest wood shavings. To secure uniformity* of temperature the back of the inner chamber was lined with thick sheet-copper. Originally the front was similarly provided, a small aperture being cut for the cathetometer readings. But after the first experiment this was found very inconvenient and was discarded. Access to the box was obtained from the front, the space between the doors of the two boxes being filled with a movable pad stuffed with shavings. The inner door occupied about half the front of the ice-chamber. With these arrangements the temperature of the interior altered very slowly, often not more than a degreee in 24 hours, though no special precautions were taken to keep the temperature of the room constant.

We were not so successful in maintaining uniformity of temperature. The minimum thermometer was hung at the back of the chamber on a level with the middle of the ice. The maximum was placed with its bulb at the bottom of the chamber at the end removed from the tin. And we often found that the temperature at the time, shown by the maximum thermometer, was one or one and a half degrees lower than that shown by the minimum. In the temperatures given in the tables allowance is made for this. We found, however, that the variations in the plasticity due to the temperature were far exceeded by others, due probably to changes in the crystalline structure of the ice. In explanation of the considerable variation of temperature occasionally recorded in the tables, we must add that, in order to raise or lower the temperature, the inner chamber was sometimes left wholly or partially open. The front of the box was close by an open window, and was generally exposed to a decidedly lower temperature than the back, so that opening the doors to take the readings would seldom raise the internal temperature materially.

The bar of ice for an experiment was roughly sawn out and then shaped more carefully with a knife. A hole was bored near each end with a hot steel knitting needle. This was found to be the only method of making a hole free from the risk of splitting the ice. In each hole was frozen a short piece of steel knitting needle with the

* Uniformity refers to space, constancy to time.

ends projecting slightly. In the later experiments we used pieces of glass tube or rod for needles, to obviate any possible exaggeration of the extension through the needle bending in the ice. The glass had the further advantage of being a bad conductor of heat. We found that, when air above freezing point entered the chamber during the taking of a reading, the steel needles were apt to work loose, although the body of the ice had not time to materially rise in temperature. Such readings are of course discarded. The two conical collars were filled with ice by freezing water therein. The upper collar was taken out and inverted, and its brass plate levelled. Then the bar was carefully placed in a vertical position and frozen on. The bar was next hung in position in the chamber and frozen on to the ice on the lower collar *in situ*.

In the first experiment the measurement of the distance between the upper and lower needles was made with a cathetometer. On the two ends of each needle were glued pieces of paper, on each of which fine ink cross lines had been drawn. The cathetometer was not of the ordinary construction and merits a short description, as, though in practice it was not very successful, in principle it has, we believe, several advantages over the ordinary form. The stand consists of a vertical rod supported by three levelling screws. On this rod slides a metal block, provided with a clamp and slow-motion screw. The telescope rests on this block, being movable through ninety degrees about a vertical axis. The bearing of the telescope is the only mechanical part of the instrument that requires special care. For the cross wires of the ordinary telescope is substituted a micrometer scale. The millimetre scale is fixed on a separate stand as near as possible to the bar of ice and at the same distance from the telescope as the ice is, and is left untouched during the observations, so that it is of no consequence, for measuring small extensions, if it be not quite parallel to the direction of the tension. The distance from the telescope to the ice or to the scale was about 30 cm. On the top of the telescope is fixed a level. We carefully adjusted this, so that when the bubble was at its zero the axis of rotation of the telescope was in the vertical plane at right angles to the tube of spirit. Then if the bubble remained in its central position in every azimuth of the telescope, we could be sure the axis of rotation was vertical.

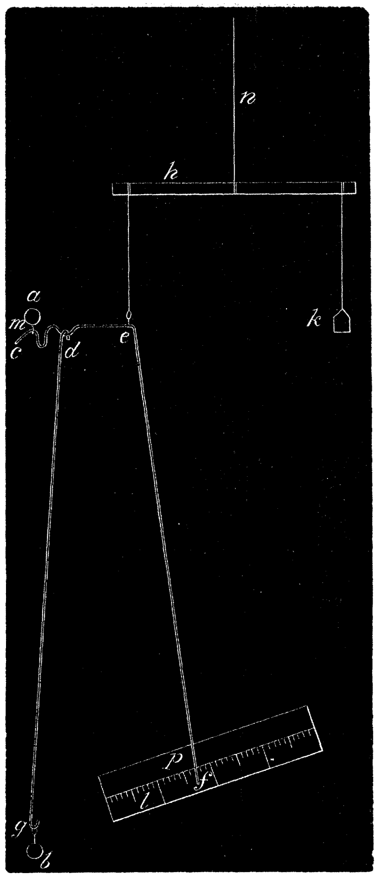
The observation was taken by reading the position, on the micrometer scale, of the image of the mark on the needle, then swinging the telescope round and reading the position, on the micrometer scale, of the two nearest divisions of the millimetre scale. By interpolation the exact height, on the millimetre scale, of the mark on the needle was then readily found. It will be noticed that the cathetometer need only remain steady while the telescope is swung round from the needle to the scale; whereas in the ordinary form there is a danger

of the whole stand being slightly displaced when the telescope is slid down to its lower position. In fact in our circumstances an ordinary cathetometer would have been practically useless, owing to the bending of the floor and table at the slightest movement of the observer. Observations, even with our special form, required the utmost care. The micrometer scale had twenty divisions, each 0·12 mm. in actual size, and corresponding to about 0·3 mm. on the other scale. The magnification of the telescope, as compared with the eye at 9 inches, was about 5. This was scarcely great enough. We intended also to have the micrometer divisions half the size, but the maker was not able to graduate it so finely. Indeed, as it was, the lines were rather too thick.

By estimating tenths of the micrometer divisions we could read to 0·03 mm., but the readings might easily be at least 0·06 mm. in error. Each determination of the length between the needles depends on four readings, the upper needle, and its corresponding scale division, and the lower needle, and its scale division. If the four readings happened to have each the maximum error 0·06 mm. with a suitable sign, the total error might be 0·24 mm. Such a combination of chances is highly improbable, but an error of 0·1 mm. is obviously not unlikely. The cathetometer would have been a useful instrument for measuring a large and regular extension with accuracy, but it was not adapted to detect very small extensions, and a system of levers, which we adopted as a rough mode of measurement in our second set of apparatus, proved so much more satisfactory and suitable to our purposes, that we almost entirely discarded the cathetometer. This contrivance is shown in fig. 3, in the form finally adopted. a and b are sections of the projecting ends of glass needles fixed in the ice, $cdef$ is a bent iron wire, "the indicator," hooked to a wire loop m securely fastened to a, h is a wooden lever suspended by a thread n, which owing to the counterpoise k, pulls the indicator upwards with a thread fastened to a wire loop at e. The indicator is kept from rising by the connecting fibre, a piece of stiff wire hooked at one end to the loop g, fastened to b, and at the other to a bend d* in the indicator. The lower end of the indicator gives the reading on a paper millimetre scale l, gummed on to the mirror p. The mirror, of course, enables the observer to avoid errors of parallax. The stand of the mirror is glued to the lower collar. To appreciate the action of the levers, regard a for the moment as fixed, then lowering b through a small distance r will move f through a distance $s = \nu r$ at right angles to mf, where ν is the ratio of the distance mf to the perpendicular let fall from m on the line gd produced if necessary. If md be made perpendicular to gd, when f is in the middle of the scale, the multiplier ν remains practically constant. This precaution was not always taken, but

* This was a deeper bend than is shown in the figure.

FIG. 3.



allowance is made for the resulting error. Two lever systems were required, one for the outer ends, and the other for the inner ends of the needles passing through the ice. In Experiment 2 we used two scales and mirrors which enabled the readings to be taken with great accuracy. Afterwards we contented ourselves with one, which gave quite sufficient accuracy for any but homogeneous ice. In the first few experiments we used glass fibres, both for the indicator and connecting fibre, as we feared some slight motion of *f* might arise from the "elastic recovery" of the wire. This was put to the test of experiment. A long piece of the same kind of wire was bent sharply at an angle, and the two ends brought nearly into contact. It was hung over a nail, and the distance between the ends measured from time to time. The effect of the gradual unbending of the angle would

in this case, owing to the greater length of the arms, be about twice as great as in the extension experiments, and yet it was found to be scarcely perceptible. For practical convenience in setting up the apparatus the wire was found immensely superior. The trouble of fixing in position a delicate arrangement of brittle glass fibres, in an awkward place like the back of the ice chamber behind the bar of ice, can hardly be realised by any one who has not tried it.

In the first few experiments the loops *m* and *g* were not used, and the indicator and connecting fibres were simply hooked over the needles *a* and *b*. And in Experiments 2, 3, 4, and 6, no efficient precautions were taken to prevent slipping along the needle. It is to be remarked, however, that any such slipping would produce an apparent contraction, and, owing to the sudden alteration of the rate of extension, any slipping of importance could hardly escape detection.* Such cases are either omitted or specially mentioned. The lever *h* and counterpoise were found rather troublesome, and will probably be dispensed with next year, by putting the connecting fibre on the other side of the needle.

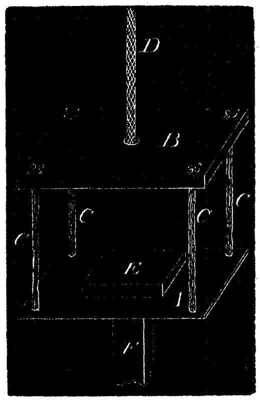
Our second apparatus, which we shall call the rough apparatus, was of much simpler construction. Instead of the collars we used two iron plates, each about 12 cm. square with a hole 2·5 cm. square in the centre. The bar to be tested was passed through the hole and frozen on to a block of ice on the other side of the plate. The upper plate was suspended by cords attached to holes at the corners, and from the lower plate was suspended by similar cords a bucket, in which various weights could be placed. In Experiments 3 and 4, the four cords were simply knotted together, and hung over an iron hook fastened to a single cord. But it was difficult in this way to ensure that the line of action of the tension should be the central line of the bar of ice, and we thought it likely that the bending in Experiment 4 was due to this cause, so we adopted the contrivance shown in fig. 4.

A is the upper iron plate, F the bar of ice attached to the block of ice E.† B is a wooden plate with holes at the corners and a hole at the centre, in exactly the same relative positions as the holes in the corners and the centre of the square hole in A. CCCC are four cords of equal length, and D the main cord by which the whole is upheld. When the arrangement is in equilibrium, the cords C will be vertical as well as the cord D, so the line of action of the tension, which is the central line of the cord D, will pass through the centre of the square hole in A, even though the two plates be not quite horizontal. The same remarks apply to a similar arrangement for the lower iron plate. If

* In almost every experiment far more readings were taken than are recorded below.

† This block was thicker than in the figure.

FIG. 4.



the bar be not attached accurately at right angles to the plates, it will take up a vertical position and the plates will be tilted. This contrivance was successful, for the icicle, which owing to its symmetrical formation would probably under uniform tension stretch equally on both sides, showed but small signs of bending. So we think it fair to conclude that in the later specimens the bending was due to their unsymmetrical structure.

In the later experiments (6, 8, 9 and 10) the apparatus was enclosed in a single box of wood about 3 cm. thick. The box was jacketed on the outside with a layer of hay about 5 cm. thick, covered with paper or felt. The cords, leading to the support and the weight, passed through holes in the top and bottom well plugged with cotton-wool. In all cases, except when the contrary is expressly mentioned, the bar of ice was wrapped in gutta-percha tissue to check the evaporation.

The polariscope was of the simplest possible form. The light transmitted by a sheet of thin paper was reflected at the polarising angle by a pile of three glass plates towards a Nicol prism supported in the same framework. With its aid it was easy to see the boundaries of the various crystals in a plate or bar of glacier ice, though not a trace of division could be detected with the naked eye, and with some difficulty the direction of the optic axes of a few of the larger crystals could be made out. In the bath ice the homogeneousness of the crystal could be readily tested, by watching the unchanged position of the rings and cross while the bar was moved across the field. In lake ice a half-inch plate, cut at right angles to the columns and viewed in the polariscope, showed a series of irregular polygons black, white, or grey, when the empty field was black. The almost invariable absence of colour proved that few or none of the

optic axes were nearly parallel to the length of the columns. That the axes, however, were not accurately perpendicular to the length of the columns, *i.e.*, horizontal in the original position on the lake, was shown by examining separate columns. After allowing the ice to thaw slightly, or better after leaving it in the rays of the sun for twenty minutes, the columns could be easily separated.

Detailed Account of the Experiments.

It will be more convenient to describe all the experiments made with Main's apparatus first, than to keep to the chronological order.

Experiment No. 1. *Main's Apparatus.*—Measurements were taken with the cathetometer. The specimen was a square bar of ice, taken from the surface of a bath of water about a foot deep, and cut into shape with a knife. It was perfectly clear and free from bubbles. It was wrapped in gutta-percha tissue, which was not removed till the end of the next experiment. The "needles" were pieces of steel knitting needle. The area of the section was 8·1 sq. cm., and the tension 3·7 kilos. per sq. cm.

Date.	Distance between needles.	Extension.	Temperature.	Difference between the two sides.	
				Upper.	Lower.
	mm.	mm.		mm.	mm.
Jan. 14, 11 h.	163·93	0·0	−3·0°	4·1	4·6
„ 16, 9 h.	164·06	+0·13	−8·0	4·5	4·3
„ 17, „	163·91	−0·02	−7·0	4·3	4·2
„ 18, „	164·04	+0·11	−6·2	4·3	4·2
„ 19, „	163·98	+0·05	−5·5	4·3	4·2
„ 20, „	164·08	+0·15	−5·0	..	..
„ „ „	164·01	+0·08	„	..	..
„ 21, „	164·13	+0·20	−4·0	4·4	4·1

The hours in the first column are reckoned from midnight. The third column gives the extension observed, measured from the length at the first reading. The fourth column gives the temperature just before each reading. The maximum temperature during the whole period was −3·0° and the minimum −8·5°. The fifth column gives roughly the difference between the heights of the marks on the right and left ends of the upper needle, and the sixth column the same thing for the lower needle. These are added to show that a slight bending took place chiefly between the 14th and 16th. On removing the gutta-percha, at the end of the next experiment, a surface crack was found which may have occurred at the same time. Each reading

on the micrometer scale of the cathetometer was taken twice, the telescope having been turned about the vertical axis in the interim. The two generally agreed. If not the mean was taken. On the 20th, however, a second set of readings was taken, the telescope having been slid down the rod in the interim. Both determinations are given.

The errors of a cathetometer reading have been already discussed. If we allow 0·11 mm. as a possible error in each determination of the length, the observations are consistent with no real extension. But taking the last two columns into consideration it seems probable that there was an extension of 0·1 mm. between the 14th and 16th and none later. Even if the total extension had been 0·2 mm., this would have corresponded to a mean extension per hour per length of 10 cm. of only 0·0007 mm.

Experiment No. 2.—The same piece of ice was fitted up with glass indicators and glass connecting fibres, the needles being the same as before. Each indicator was provided with a mirror and scale set close up to it, so readings on the scale could be taken to 0·2 mm. But on the other hand there was a possibility of the indicators slipping on the needles and thus occasioning a slight apparent contraction. The multiplication on the outer side was 34, on the inner 26. Thus an extension of 0·007 mm. could be detected. The second, third, and fourth columns of the following table give the extensions, measured from the length at the time of the third observation (for a reason mentioned below), and reduced to the proportionate amount for a length of 10 cm. They are probably correct to 0·004 mm.

Experiment No. 2.—Main's Apparatus. Bath Ice. Length between Needles 16 cm. Tension 4·9 kilos. per square centimetre.

Date.	Extension per 10 cm.			Temperature at the time.
	Outer.	Inner.	Mean.	
	mm.	mm.	mm.	
Jan. 30, 10 h. 30 m.	0·016	0·000	0·008	− 5·0°
,, ,, 16 h. 30 m.	0·016	0·055	0·035	..
,, 31, 9 h. 15 m.	0·000	0·000	0·000	−15·0
,, ,, 16 h. 30 m.	−0·002	0·000	−0·001	−12·5
Feb. 1, 9 h. 30 m.	0·000	0·019	0·009	− 8·5
,, 2, ,, ,, ...	−0·002	0·002	0·000	− 8·8
,, ,, 16 h.	0·007	0·022	0·014	− 6·4
,, 3, 9 h. 45 m.	0·007	0·022	0·014	− 6·5
,, 4, 12 h.	0·009	0·045	0·027	− 3·7
,, ,, 18 h. 10 m.	0·007	0·050	0·028	− 1·6
,, 5, 9 h. 30 m.	0·007	0·048	0·027	− 1·5
,, ,, 16 h.	0·007	0·048	0·027	− 1·0

The temperature on the afternoon of the 30th was not taken, but the notebook contains a statement that it was colder than the morning.

Since the box was left open all night, the temperature given by the thermometer on the morning of the 31st may well have been rather lower than that of the ice. Between the 1st and 2nd, an apparent contraction of 0·017 mm. on one side took place without change of temperature. This looks as if the indicator had slipped. Making allowance for these, the mean extension from the 31st, 9 h. 15 m., to the 5th, 16 h., follows the temperature very fairly, considering the uncertainty of the latter. We have arranged the table to show this. But during the first six hours there was an expansion on one side of 0·088 mm. in actual magnitude, which we attribute to a slight yielding at the crack. Counting the contraction as a slip, and making no allowance for temperature, the mean rate during the whole 150 hours was 0·00019 mm. per hour per length of 10 cm.

If we suppose that the extension during the last five days was entirely due to temperature, and that the coefficient of expansion of the glass of the connecting fibre was 0·000009, we have between $-12{\cdot}5°$ and $-8{\cdot}5°$ a coefficient of linear expansion of ice of 0·000034, between $-8{\cdot}5°$ and $-3{\cdot}7°$ of 0·000060, and between $-3{\cdot}7°$ and $-1{\cdot}0°$ of 0·000009.

Into the complicated question of the expansion of ice with temperature we do not care to enter fully. We will merely cite two investigations. The best observations on the cubical coefficient seem to be those of Pettersson ("On the Properties of Water and Ice," '*Vega* Expedition,' vol. 2, Stockholm, 1883). We deduce from his figures the corresponding linear coefficients, supposing ice to be isotropic in this matter. With ice from ordinary distilled water he obtained 0·000053 between $-12°$ and $-2°$. This ice began to contract at some point between $-0{\cdot}35°$ and $-0{\cdot}25°$. With ice from the purest water he could obtain, the coefficient rose from 0·000055 between $-17°$ and $-10°$ to 0·000057 between $-4°$ and $-3°$, and then decreased, till it changed sign at some point between $-0{\cdot}15°$ and $-0{\cdot}03°$. Ice containing 0·014 per cent. of chlorine, in the shape of salts, began to contract at $-2{\cdot}5°$. In these experiments the water was frozen in the dilatometer, so there was no chance of the impurities being expelled by the process of solidification as in the case of ice formed slowly on the surface of some depth of water. His purest water, however, was so good as to be seriously affected by boiling for a short time in a clean glass vessel.

The coefficient of linear expansion has been determined directly by Andrews ('Roy. Soc. Proc.,' June, 1886). He found 0·0000505 between $-18°$ and $-9°$, and 0·0000735 between $-9°$ and $-0°$. It is possible that the difference between the determinations of these two

experimentalists is owing to an unequal expansion of ice in different directions. At any rate, taken together, they are sufficient to explain our rough results, on the supposition that the extension of the last five days was entirely due to the rise of temperature.

The experiment was brought to a close by the bar breaking at a point above the upper needle, where it was not protected by gutta-percha tissue, and had become very thin through evaporation. The thickness had been reduced by this cause in three weeks from 2·85 cm. to 2·2 cm. The temperature, at which the fracture occurred, was between −0·5° and −1·0°, certainly not above the former. The breaking tension was 8·35 kilos. per sq. cm. There was a groove running right round the bar near the middle of its length, but no sign of a crack could be seen in the interior of the ice. This groove may have been caused by the outer layer cooling more rapidly than the interior. Under the polariscope no break in the continuity of the crystalline structure could be detected. The rings and cross were seen very plainly, and the direction of the optic axis appeared to be the same on both sides of the crack. It was perpendicular to the length of the bar and also to the needles. By a rough measure of the rings we found the difference between the two indices of refraction to be 0·0018. In quartz it is 0·0094; in Iceland spar 0·172.

Experiment No. 5. *Main's Apparatus.*—The specimen was a piece of glacier ice (B). The measurements were taken with the cathetometer. We had already found, in the other apparatus, that glacier ice would stretch, but we thought it desirable to confirm the fact with a different mode of measurement. So in this one case we used the cathetometer again, in spite of its disadvantages for this kind of work. The length between the needles was about 20 cm., the area of section 7·3 sq. cm., and the tension 2·7 kilos. per sq. cm. The second

Glacier Ice B. Length between Needles 20 cm. Tension 2·7 kilos. per sq. cm.

Date.	Temperature at the time.	Interval.	Extension.	Rate per hour per 10 cm.	Temperature.	
					Max.	Mean.
		hours.	mm.	mm.		
Feb. 9, 9 h...... ,, 10, ,, ...	−2·5° −2·5	24	0·78	0·0160	−2·5°	−3·5°
,, ,, 16 h. 30 m. ,, 11, 8 h. 45 m.	.. −3	16·25	0·44	0·0135	−2·5	−4·5
,, 12, ,, ,, 13, 16 h. 45 m.	−4 −0·5	32	0·53	0·0083	−0·5	−3·0
Total.........	..	72·25	1·75	0·0116	..	..

column gives the temperature just before each observation, the fourth the actual extension during the interval in millimetres, the error probably not exceeding 0·1 mm., the fifth the rate per hour per length of 10 cm., and the two last the maximum and mean temperatures during the interval.

On the 10th and 11th the ice broke at the collar, and had to be frozen together again. It will be noted that the rate of extension decreases with the time, more than can be explained by errors of observation, though the tendency of the temperature is to rise.

Experiment No. 7. Main's Apparatus.—The same piece of ice was used, cut a little shorter (glacier ice C), and fitted with wire indicators. Only one scale was used for the two indicators, so the readings cannot be trusted beyond 0·5 mm. on the scale. As the multiplication was generally about 16, this gives an error in the actual extensions, when small, not greater than 0·03 mm. When the extensions are large the error is greater, owing to an uncertainty of perhaps 10 per cent. in the multiplication. The "needles" were glass tubes. The length between the needles was 18 cm., and the area of section 7·3 sq. cm. The first column gives the time of each reading, the second the temperature at that time, the third the interval between two readings, the fourth and fifth the extension shown by the outer and inner indicators, the sixth the mean rate of extension per hour per length of 10 cm., the seventh the tension, the eighth, ninth, and tenth the maximum, minimum, and mean temperatures, during that interval.

On the 17th February the tension was increased by one-half, and the ice in consequence broke at the collar. It was frozen in again, and the tension reduced to the original value. On the 8th March an hour was occupied in readjusting the wire indicators. The sixth column shows a rapid decrease of speed for the first five days, followed by fluctuations due apparently mainly to the temperature, the rate at $-4°$ being about double that at $-9°$. An addition of one-half to the tension increased the rate 500 per cent. for the first two days of the change. This increased rate in its turn showed a tendency to sink, more or less counterbalanced by the rising temperature. The fourth and fifth columns show the curious way in which the more rapid extension alternates from one side to the other.

This piece of ice, taking the two experiments together, was under tension for twenty-five days, and extended altogether about 6 mm., *i.e.*, about 3 per cent. of its length. At the close of the experiment the divisions between two or three of the crystals at one point of the bar almost amounted to cracks, and at that point there was a decided twist in the bar, estimated at 10°. There were a great many bubbles in the ice, and the crystalline structure was very complex. There was no particularly large crystal.

Glacier Ice C. Length between Needles 18 cm.

Date.	Temperature at the time.	Interval.	Extension. Outer.	Extension. Inner.	Rate per hour per 10 cm.	Tension kilos. per sq. cm.	Temperature. Max.	Temperature. Min.	Temperature. Mean.
		hours.	mm.	mm.	mm.				
Feb. 16, 10 h.	−5·0°								
" 17, 9 h.	−7·3	23	0·25	0·31	0·0068	2·55	−2·5°	− 7·3°	−4·5°
" " 15 h. 15 m.	−5·8								
" 18, 9 h. 30 m.	−8·5	18·25	0·19	0·09	0·0042	"	−5·5	− 8·5	−7·0
" 21, 8 h. 45 m.	−6·0	71·75	0·09	0·25	0·0013	"	−6·0	−12·0	−9·0
" 23, "	−8·5	48	0·14	0·12	0·0015	"	−4·8	− 8·5	−7·0
" 25, 9 h.	−7·2	48·25	0·07	0·19	0·0015	"	−7·2	− 8·8	−7·8
" 27, 9 h. 15 m.	−4·5	48·25	0·15	0·22	0·0021	"	−4·0	− 7·2	−5·0
" 29, 9 h.	−4·5	47·75	0·32	0·19	0·0029	"	−2·8	− 4·5	−3·8
Mar. 2, 8 h. 45 m.	−6·5	47·75	0·21	0·12	0·0018	"	−4·5	− 6·5	−5·6
" 4, 9 h.	−5·2	48·25	0·97	0·94	0·0110	3·85	−3·5	− 6·5	−5·2
" 6, "	−7·0	48	0·55	0·62	0·0068	"	−5·2	− 7·5	−6·5
" 8, "	−5·7	48	0·56	0·78	0·0078	"	−5·0	− 7·0	−6·0
" " 10 h. 15 m.									
" 9, 16 h. 30 m.	−2·4	30·25	0·27	0·72	0·0090	"	−2·4	− 5·7	−3·8
Total..........	..	527	3·8	4·55	0·0044	..	..	..	..

We now come to the experiments made with the rough apparatus. At first it fully deserved the name, but later on, viz., in Experiments 6, 8, 9, and 10, the results were quite as trustworthy as in the more elaborate arrangement.

Experiment No. 3. *Rough Apparatus.*—The specimen was a circular cylinder frozen in Main's mould, about 20 cm. between the needles.

The area of section was 6 sq. cm., and the tension 4 kilos. per sq. cm. The measurement was taken with glass indicators. A long straight glass fibre was used as indicator, bent at one end to hook under the lower needle, and supported in a nearly horizontal position by a glass connecting fibre hooked over the upper needle. The vertical scale was attached to an arm projecting from the upper iron plate.

During the first 22·5 hours the ice extended 3·7 mm. on the outer side, and contracted 0·75 mm. on the inner side. During a subsequent six hours it extended 1·7 mm. on the outer side and 0·6 mm. on the inner. The mean rate per hour per length of 10 cm. was therefore 0·046 mm. The temperature is not known with any certainty. This ice was never examined under the polariscope, but owing to the mode of formation described fully at the end of the paper, we may be certain the structure was in the highest degree irregular. It was probably, however, tolerably symmetrical about the axis, so the bending may be attributed to the eccentric application of the pull.

Experiment No. 4. *Rough Apparatus.*—The specimen was a piece of glacier ice (A), composed of perhaps a dozen "grains" very irregularly arranged, the axes of some being at right angles, of others parallel, to the length. Distance between needles about 22 cm. The area of the section is a little uncertain, as it was not measured *in situ*, and the ice was not protected from evaporation. It may be taken as 6·5 sq. cm., and the tension as 1·66 kilo. per sq. cm. The ice was subjected to tension for about eighty-five hours altogether, but we only give the results for the last twenty-seven, as at first the indicators appear to have slipped, and, after precautions had been taken to prevent slipping the two indicators happened to come in contact. The indicators were arranged as in the last experiment, but the readings were improved by attaching a mirror to the scale. The multiplication was about 30, and the extensions may be trusted to 0·03 mm. The first column in the annexed table gives the time of each reading, the second the temperature at that time, the third the interval between two readings, the fourth and fifth the actual extensions measured by the outer and inner indicators in that interval, and the sixth the rate per hour per length of 10 cm.

The temperatures are somewhat uncertain, as the ice was not enclosed in a box, and the temperature of the room was very far from being uniform. The last four temperatures were taken by a thermometer hung close by the ice and on the same level. The minimum of the night by this thermometer was −3·3°. The high temperature at 21 h. 15 m. was due to the window of the room having been nearly closed. It was then thrown wide open, so the temperature must have soon fallen again. So the interval before this reading, 0·0°, would probably be much warmer on the average than the subsequent

Glacier Ice A. Length between Needles, 22 cm. Tension 1·66 kilos. per sq. cm.

Date.	Temperature.	Interval.	Extension.		Rate per hour per 10 cm.
			Outer.	Inner.	
		hours.	mm.	mm.	mm.
Feb. 3, 9 h.	−2·5°				
		3·75	0·23	0·02	0·015
„ „ 12 h. 45 m.	−1·0				
		3·75	0·19	0·01	0·013
„ „ 16 h. 30 m.	−4·0				
		4·75	0·35	0·10	0·022
„ „ 21 h. 15 m.	0·0				
		11·25	0·53	0·16	0·014
„ 4, 8 h. 30 m.	−3·0				
		3·50	0·19	0·08	0·018
„ „ 12 h.	−1·0				
Total	..	27·0	1·49	0·37	0·0156

interval. Thus the sixth column shows that the ice became more plastic as it neared the thawing point. The unequal extensions in the fourth and fifth columns may well have been due to eccentric application of the tension.

Experiment No. 6. *Rough Apparatus.*—The specimen was an icicle trimmed with a knife to an uniform circular section. The apparatus was greatly improved. The new mode of suspension was adopted, specially arranged, as described above, to ensure the tension acting along the central line of the bar. The indicators were hooked over the top needle and bent at right angles so as to point downwards, as in Main's apparatus. They were of glass, and no thoroughly efficient means was taken to prevent slipping along the needle, but we do not think any slipping can have taken place during the observations quoted below. The whole apparatus was enclosed in a jacketed box—which was, however, generally left open at night—and a centigrade thermometer, graduated to tenths, was hung in the box on a level with the middle of the ice.

In the table the fourth and fifth columns give the actual extensions during each interval, which may be trusted to 0·015 mm., and the sixth column the mean rate of extension per hour per length of 10 cm. The second column gives the reading of the thermometer at the time of the observation, and the last two columns the maximum and mean temperatures of the ice during each interval. These are tolerably accurate, as many observations were taken besides those here quoted. The ice was not protected from evaporation, so the

Icicle. Length between Needles, 19 cm.

Date.	Temperature at the time.	Interval.	Extension.		Rate per hour per 10 cm.	Tension, kilos. per square centimetre.	Temperature.	
			Outer.	Inner.			Maximum.	Mean.
		hours.	mm.	mm.				
Feb. 11, 22 h.	−2·0°							
		10·5	0·035	0·070	0·0026	2·0	−2·0°	−2·3°
,, 12, 8 h. 30 m.	−2·7							
		7·75	0·0	0·045	0·0015	2·1	−0·7	−1·3
,, ,, 16 h. 15 m.	−0·7							
		16·5	0·070	0·105	0·0037	2·1	0·0	−0·2
,, 13, 8 h. 45 m.	+0·5							
		11·75	..	..	..	0	0·0	0·0
,, ,, 20 h. 30 m.	+0·5							
		13·0	0·100	0·0	0·0024	2·3	0·0	−1·5
,, 14, 9 h. 30 m.	−3·0							
		7·0	0·075	0·020	0·0035	2·4	−0·7	−2·0
,, ,, 16 h. 30 m.	−0·7							
		16·5	0·165	0·070	0·0032	2·5	0·0	−0·8
,, 15, 9 h.	+0·6							
		4·75	0·035	0·040	0·0041	2·6	0·0	0·0
,, ,, 13 h. 45 m.	+0·5							
Total	..	78	0·48	0·36	0·0028	..	..	..

section gradually diminished, and the tension consequently increased, as given in the seventh column. The mean section was about 4·1 sq. cm. The ice was under tension for twenty-four hours previous to the observations given below, but during this time the indicators seem to have slipped.

The weight was removed for twelve hours on the 14th owing to the thaw. It is curious to notice how irregularly the extension is divided between the two sides; the ice bends first one way then the other. The fluctuations in the mean rate of extension seem mainly due to the temperature. During thirteen hours at a temperature between $-1·5°$ and $-3·0°$ the rate was 0·0028, while during thirty-eight hours at a temperature above $-0·7°$ the rate was 0·0034. The ice was full of minute bubbles, though not in sufficient quantity to make it quite opaque. The component crystals were very small, less than a millimetre in diameter, and with optic axes arranged quite irregularly.

Experiment No. 8. *Rough Apparatus.*—The specimen was a piece of glacier ice (D). The wire indicators and connecting fibres were hooked through wire loops firmly fastened to the glass needles embedded in the ice, so there was no possibility of slipping. The multiplication was about 22, so the small extensions are accurate to 0·02 mm. The area of section was 6·3 sq. cm. The table is arranged as in the last experiment (6).

Thus the whole extension in three and a half days was more than 4 per cent. of the length. At 20 h. 15 m. the inner indicator had moved off the scale against a stop, so the extension was probably rather greater, certainly not less than that given. The extension at a particularly low temperature, mentioned in the general summary, was between February 18th, 21 h., when the temperature was $-9·0°$, and February 19th, 9 h. 15 m. There was a contraction on the outer side during this interval of 0·01 mm., and an extension on the inner side of 0·23 mm., so the mean rate per hour per 10 cm. was 0·0065 mm.

It should be mentioned that the points on the glass needles, where the indicators were attached, were not quite close to the ice, but at the distance of a centimetre perhaps. Hence, while the mean rate is correctly given, the extension on the inner side of the bar is exaggerated, and that on the outer side made too small. Taking the ice as 2·5 cm. thick, this consideration leads to the result that the total extension of the outer face of the bar was 2·9 mm., of the inner face 9·7 mm.

This experiment shows how completely the plasticity depends on changes in the internal structure of the ice. Thus, for the first two days we find, under a slight stress, a moderate rate showing some tendency to decrease more rapidly than can be easily attributed to the fall of temperature. An increased tension produces as usual a

Glacier Ice D. Length between Needles, 14 cm.

Date.	Temperature at the time.	Interval.	Extension.		Rate per hour per 10 cm.	Tension, kilos. per sq. cm.	Temperature.	
			Outer.	Inner.			Max.	Mean.
		hours.	mm.	mm.				
Feb. 18, 10 h.	$-6.7°$							
,, ,, 17 h.	-6.0	7·0	-0.03	0·27	0·0122	1·45	$-5.0°$	$-6.0°$
,, 19, 9 h. 15 m.	-11.0	16·25	-0.04	0·28	0·0054	,,	-6.0	-10.0
,, ,, 19 h. 15 m.	-7.7	10·0	$+0.01$	0·19	0·0071	,,	-7.0	-8.0
,, 20, 9 h.	-6.5	13·75	-0.03	0·32	0·0075	,,	-6.0	-7.0
,, ,, 12 h. 45 m.	-5.4	3·75	-0.06	0·33	0·026	2·55	-5.4	-6.0
,, ,, 14 h.	-4.7	1·25	-0.01	0·37	0·103	,,	-4.7	-5.0
,, ,, 16 h. 30 h.	-4.2	2·50	-0.08	0·99	0·128	,,	-4.2	-4.5
,, ,, 20 h. 15 m.	-4.6	3·75	-0.09	1·20?	0·105?	,,	-4.2	-4.4
,, 21, 8 h. 45 m.	-3.7	12·5	$+0.06$	0·29	0·010	1·03	-3.7	-4.2
,, ,, 9 h. 15 m.	-3.7	0·5	-0.01	0·33	0·228	2·50	-3.7	-3.7
,, ,, 10 h.	-3.0	0·75	$+0.08$	0·95	0·485	,,	-3.0	-3.3
,, ,, 11 h.	-2.9	1·0	$+0.15$	1·68	0·65	,,	-2.9	-3.0
,, ,, 11 h. 20 m.	-2.2	0·33	$+0.39$	1·10	1·58	,,	-2.2	-2.5
,, ,, 11 h. 30 m.	-2.0	0·16	$+0.12$	0·77	1·88	,,	-2.0	-2.1
,, ,, 12 h. 15 m.	-2.0	0·75	$+0.05$	0·70	0·35	1·80	-2.0	-2.0
,, ,, 13 h. 45 m.	-1.7	1·5	0·0	1·10	0·265	,,	-1.7	-1.7
,, ,, 16 h.	-2.0	2·25	-0.06	0·77	0·110	,,	-1.7	-1.8
,, ,, 21 h. 30 m.	-3.7	5·5	-0.16	0·83	0·043	,,	-2.0	-2.8
Total	..	83·5	$+0.21$	12·47	..	..	..	..

large increase in the velocity. But it has further the remarkable effect of transforming a slow retardation into a rapid acceleration. A light tension now reduces the velocity to nearly the old figure. But as soon as the former tension is restored, the acceleration continues till the velocity reaches nearly 2 mm. an hour. It is true that this acceleration was attended by a rising temperature, but it seems far too great to be attributed to that alone. We may fairly conclude that the process of extension itself has sometimes the effect of increasing the apparent plasticity. Reducing the tension by one-third brought down the velocity at once by four-fifths, and, strange

to say, impressed a gradual retardation in spite of a rising temperature. It would thus appear that in this case, while a rapid extension increased the plasticity, a gradual extension had the effect of diminishing it. This is an anomalous result, but it must be remembered that we are measuring the sum of a large number of independent actions. The behaviour of the whole is probably much more complicated than that of any one of the individuals.

Being curious to see the effect of great tension, we applied 4·2 kilos. per sq. cm. This brought the experiment to an end, for after half a minute the ice gave way. It was found broken both at the lower collar and at a point below the upper needle, where we had previously noticed a crack extending part of the way across the bar. At which point it broke first we cannot say. The bar was examined at the end of the experiment. It was nearly straight in spite of one side having extended so much more than the other. It contained several large bubbles, one perhaps 2 cm. long, drawn out into very irregular shapes, which seemed to show this piece had suffered great distortion while it still formed part of the glacier. It contained part of a very large crystal which composed, perhaps, one third of the whole bar, and ran three quarters of the length between the needles. This crystal occupied one of the angles adjacent to the inner face, which extended so much. Its optic axis was inclined at perhaps 70° to the length of the bar.

Experiment No. 9. Rough Apparatus.—The specimen was a bar of lake ice, with the crystalline columns parallel to the length of the bar. The section was 8 sq. cm. in area. The arrangements were the same as in the last experiment (8). The extensions are so small that the deduced rate during each interval would be very inaccurate. We have therefore given, in the second, third, and fourth columns of the table, the extensions measured from the length at the time of the first reading and reduced to the proportionate value for a bar 10 cm. long. They are probably correct to 0·01 mm. The fifth column gives the temperature shown by the thermometer at each reading; and the next three the maximum, minimum, and mean temperatures of the ice during each interval, estimated from a large number of observations not quoted.

Previously to 15 h., February 28th, the ice must have been thawing, probably for about an hour. The weight was removed for the next three hours. The total extension during 208 hours per length of 10 cm. was 0·145 mm. on the outer side, and 0·048 mm. on the inner, giving a mean rate per hour of 0·00046 mm. The mean rate during the first 168 hours was 0·00039 mm., and during the last 40 with the heavier weight 0·00076 mm., notwithstanding a slightly lower mean temperature. But these rates were so small as to be beyond our means of accurate measurement.

Lake Ice parallel to Columns. Length between Needles, 16 cm.

Date.	Extension per 10 cm.			Temperature.				Tension kilos. per sq. cm.
	Outer.	Inner.	Mean.	At the time.	Maximum.	Minimum.	Mean.	
	mm.	mm.	mm.					
Feb. 23, 14 h.	0·0	0·0	0·0	−5·0°				
					−4·0°	−8·0°	−7·0°	2·1
„ 26, 9 h.	0·036	0·017	0·026	−4·0				
					−2·0	−4·5	−3·2	2·1
„ 28, 8 h.	0·072	0·036	0·054	−4·1				
					0·0	−4·1	−2·0	2·1
„ „ 15 h.	0·085	0·024	0·054	+2·0				
								0·0
„ „ 18 h.	0·085	0·024	0·054	−1·0				
					−1·0	−9·0	−5·0	2·1
*Mar. 1, 17 h.	0·108	0·024	0·066	−5·0				
					−4·0	−7·0	−5·5	2·8
„ 3, 9 h.	0·145	0·048	0·096	−6·0				

* There were 29 days in February this year.

Examining the bar at the end of the experiment, we counted about thirty columns in a section, most of which ran the full length of the bar. The largest had a sectional area of about 35 sq. mm.

Experiment No. 10. *Rough Apparatus.*—The specimen was a bar of lake ice, with the crystalline columns running obliquely across at an angle of 45° to the length of the bar. The area of section was 5·5 sq. cm. The indicators, &c., were arranged as before. The temperature at the time of observation, and the minimum temperature were observed; the maximum and mean temperatures are estimated. The fourth and fifth columns give the actual extension during each interval. They are probably correct to 0·02 mm., as the multiplication was 35.

The rate shows a decided tendency to decrease, only slightly checked by the rise of temperature. The glass needles were put at right angles to the columns as well as to the length of the bar.

Lake Ice oblique to the Columns. Length between Needles, 11·5 cm. Tension, 2·75 kilos. per sq. cm.

Date.	Temperature at the time.	Interval.	Extension.		Rate per hour per 10 cm.	Temperature.		
			Outer.	Inner.		Maximum.	Minimum.	Mean.
		hours.	mm.	mm.				
Mar. 5, 9 h. 30 m. …	−6·0°							
		6·5	0·09	0·42	0·034	−5·6°	−6·1°	−5·8°
„ „ 16 h. ………	−5·6							
		17	0·12	0·42	0·014	−5·6	−7·8	−6·7
„ 6, 9 h. ………	−7·8							
		8	0·10	0·16	0·014	−5·6	−7·8	−6·7
„ „ 17 h. ………	−5·6							
		16	0·10	0·26	0·010	−5·6	−6·7	−6·0
„ 7, 9 h. ………	−6·7							
		9	0·12	0·14	0·013	−3·3	−6·7	−5·0
„ „ 18 h. ………	−3·3							
		15	0·17	0·21	0·011	−3·3	−5·8	−4·6
„ 8, 9 h. ………	−5·6							
		8	0·14	0·11	0·014	0·0	−5·6	−2·8
„ „ 17 h. ………	0·0							
Total ………	..	79·5	0·85	1·72	0·015	..	..	..

We shall now describe the experiments on compression. An oblong piece of thick plate glass was laid on the table, and on it were placed three square blocks of ice, at the angles of an equilateral triangle about 9 cm. in the side. On the ice was laid a second piece of plate glass similar to the first, and pressure applied by means of a lever at a point immediately over the centre of the triangle. Measurements were taken with callipers of the distance between the plates at three points on the edge, such that each point lay on a line through the centre and one angular point of the triangle. By drawing a diagram to scale, it was not difficult to deduce from these measurements the yielding of each block of ice. To prevent slipping, we found it necessary and sufficient to freeze a slip of paper on each end of a block of ice. A maximum thermometer was placed on the table close by the plates, and covered over with the same cloth, so that it probably gave the temperature of the ice within a degree. The horizontal section of each block was 7·5 sq. cm. in area. The fourth, fifth, and sixth columns give the actual contraction of the blocks during each interval. They are correct probably within 0·02 mm. Each measurement with the callipers was repeated, and the two readings seldom differed more than 0·02 mm.

Pressure had been applied for one day previous to those here given, but owing to an accident, its magnitude was rather uncertain. The remarkable difference between the plasticity of three specimens of glacier ice is well shown, though in this case all three pieces were from the same lump. After the experiment they were examined under the polariscope. All three were composed of smallish grains averaging perhaps 7 mm. in diameter. The increase of plasticity for a rise in temperature from $-6°$ to $-3°$ is very striking in all three pieces.

Experiment No. 2 *on Compression.*—In this three pieces of lake ice were arranged as in the last experiment. The crystalline columns were vertical, so that the pressure was applied in a direction parallel to them. The horizontal section of each piece was 7 sq. cm. The fourth, fifth, and sixth columns of the table give the contractions during each interval, calculated from the readings actually taken, as explained in the description of the last experiment. They are probably accurate to 0·02 mm. It may be mentioned that the totals are calculated to an extra place of decimals, which explains the slight discrepancy observable.

Compression Experiment No. 1. Three pieces of Glacier Ice. Initial Length, 2·9 cm. Pressure 3·2 kilos. per sq. cm.

Date.	Temperature at the time.	Interval.	Contraction.			Temperature.	
			E.	F.	G.	Maximum.	Mean.
		hours.	mm.	mm.	mm.		
Feb. 21, 9 h.	− 4·3°						
		24	0·37	0·30	0·06	−2·8°	−6·4°
„ 22, „	−12·0						
		24	0·05	0·09	0·02	−4·0	−7·0
„ 23, „	−11·4						
		24	0·09	0·17	0·00	−4·4	−7·5
„ 24, „	− 9·4						
		24	0·06	0·10	−0·02	−4·7	−6·4
„ 25, „	− 6·4						
		24	0·61	1·24	0·18	−2·8	−4·0
„ 26, „	− 2·8						
Total..........	..	120	1·18	1·92	0·25	..	..
Rate per hour per 10 cm. { 2nd, 3rd, and 4th days..............			0·0096	0·0172	0·000	..	−7·0
Rate per hour per 10 cm. { 5th day..........................			0·088	0·178	0·026	..	−4·0

Compression Experiment No. 2. Lake Ice compressed along the Columns. Length, 3·4 cm. Pressure, 3·7 kilos. per sq. cm.

Date.	Temperature at the time.	Interval.	Contraction.			Temperature.	
			A.	B.	C.	Maximum.	Mean.
		hours.	mm.	mm.	mm.		
Mar. 3, 22 h.	−3·6°						
„ 4, 9 h.	−8·3	11	0·00	0·02	0·02	−3·9°	−6·0°
„ 5, „	−6·7	24	0·01	0·02	0·01	−6·7	−7·0
„ 6, „	−7·8	24	0·00	0·00	0·01	−4·7	−6·0
„ 7, „	−5·6	24	0·00	0·00	0·01	−4·2	−5·3
Total..........	..	83	0·005	0·035	0·050	..	..
Rate per hour per 10 cm..........	..	..	0·0002	0·0012	0·0018	..	..

Thus the yielding of one piece was well within the errors of observation, of the other two only just perceptible with the instrument employed, and this small yielding may well have taken place entirely in the thin layer of irregular ice with which the paper was attached.

In the early part of the winter we made, as already mentioned, a large number of experiments on obtaining ice in the mould* free from air bubbles. We were ultimately successful, and, though our experiments proved to be of little use for their immediate object, they are of some permanent interest as tests of various methods of obtaining air-free water, so we shall describe a few typical ones. Main, the previous winter, boiled the water and let it freeze, then melted it in the mould, boiled it, and let it freeze again. The result was clear ice, except for "a small core of minute bubbles up the axis of the cylinder." By Main's advice we procured an air-pump adapted to exhaust the air from the mould. Between the pump and the mould was a good stopcock, which would maintain the vacuum for several hours. When in good order the pump would boil water at 40° C., or below. We found that this degree of exhaustion was far from removing all the air, even when applied for five hours. Boiling for half an hour, cooling *sub vacuo*, and freezing at atmospheric pressure under oil was more successful, but not satisfactory. We froze the water at atmospheric pressure to make the bubbles small, having placed a layer of oil on the top to prevent air entering. The next method proved much more effectual. We kept water *sub vacuo* for twenty-four hours at about 70° C., and let it cool *sub vacuo*, only admitting air after the freezing had begun. There were a few exceedingly small bubbles visible at one end of the rod of ice. Thawing this *sub vacuo* and keeping it again for twenty-four hours *sub vacuo* at 70° C., we got rid of the last traces of air in the rod, though there were a few in the large cone of ice.

[We conclude that, to free water from air, it should first be boiled till most of the dissolved air has escaped, and then left for a considerable time without permitting any air to have access to its surface. Boiling should be repeated at intervals to remove the air, which gradually escapes from the water and mingles with the aqueous vapour in the space above. It is probable that a high temperature quickens the process.—July 6, 1888.]

The utter irregularity of the crystalline structure of the mould ice is an obvious consequence of the mode of formation. The first ice formed, no doubt, is a layer on the surface, but the centre of this is soon broken though by water forced up from below, owing to the expansion in freezing. So what we observed in its various stages was

* This was the iron mould used by Main to form a round column of ice 2·8 cm. in diameter and 24 cm. in length, with a conical expansion at the lower end of perhaps half the volume of the column.

a ring of ice formed at the surface, which gradually extended down the sides and towards the centre, till we had a long tube of ice thinning out towards the lower end joined on to a case of ice, lining the inside of the cone. The tube grew thicker and thicker, till it became a solid bar. When a piece of sheet india-rubber was laid on the surface (to prevent air entering), it was frozen firmly to the sides of the mould, while the centre was pushed upwards into the shape of a bee-hive, till at last it burst. It was curious to find the india-rubber with the middle part drawn out into a long tube with torn edges, firmly imbedded in the ice at some little distance from the end.

In conclusion, we wish to express our thanks to Dr. Main for the use of his special stretching machine, and of the various thermometers, callipers, and much other apparatus, which he has generously placed at our service.

In case any reader of this paper should be kind enough to offer us any useful suggestions, or on the other hand should desire further information on any point, we give here the permanent address of one of the authors, James C. McConnel, Brooklands, Prestwich, Manchester, England. We may add that copies of papers bearing on the subject would be particularly acceptable.

V. "On the Organisation of the Fossil Plants of the Coal-measures. Part XV." By W. C. WILLIAMSON, LL.D., F.R.S., Professor of Botany in the Owens College, Manchester. Received June 13, 1888.

(Abstract.)

The author describes and figures a series of specimens which throw new light upon Corda's two genera *Zygopteris* and *Anachoropteris*, as they are adopted by M. Renault, but which specimens show that both these genera can no longer be retained, even by those who approve of such multiplications of ill-defined genera. He proposes, therefore, the abandonment of *Anachoropteris* and the retention of *Zygopteris*, so that "*Zygopteroid*" may be employed as a descriptive adjective in connexion with some specially remarkable forms of petiolar vascular bundles. Under the name of *Rachiopteris hirsuta*, a new group of freely branching stems or rhizomes are figured and described, characterised by having the exterior of their bark abundantly clothed, especially in what appear to be the younger shoots, with remarkably large curved multicellular hairs, closely resembling those similarly located in the young shoots of the Marsileæ; numerous cylindrical roots radiate from these axial organs. Under the provisional name of *Rachiopteris verticillata* attention is also

FIG. 1.

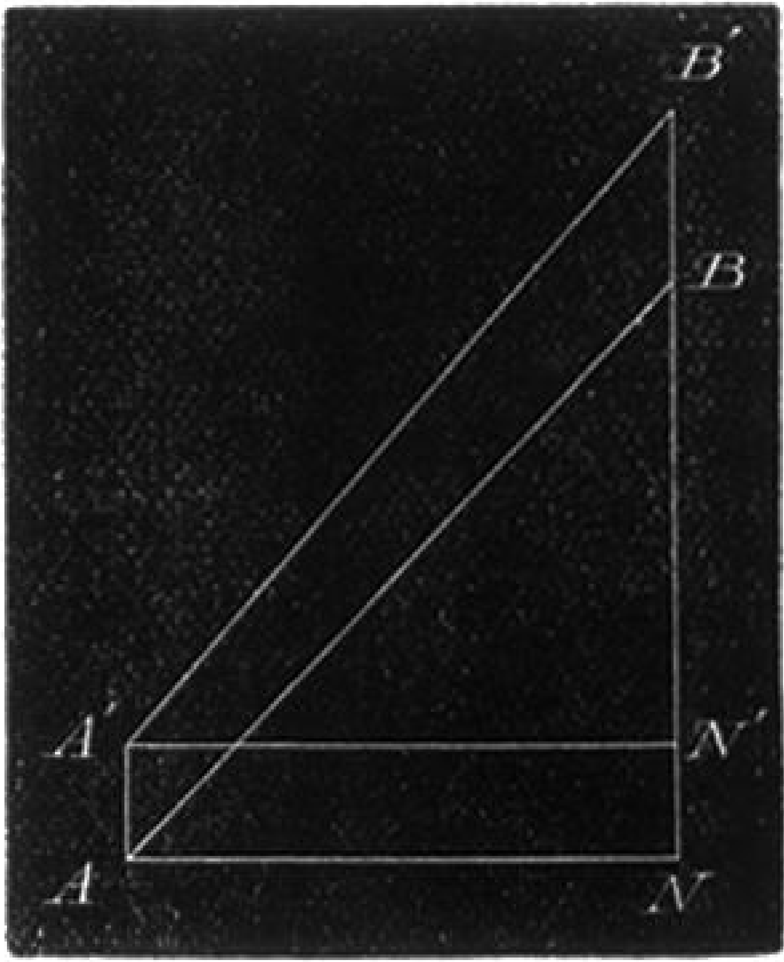


FIG. 2.

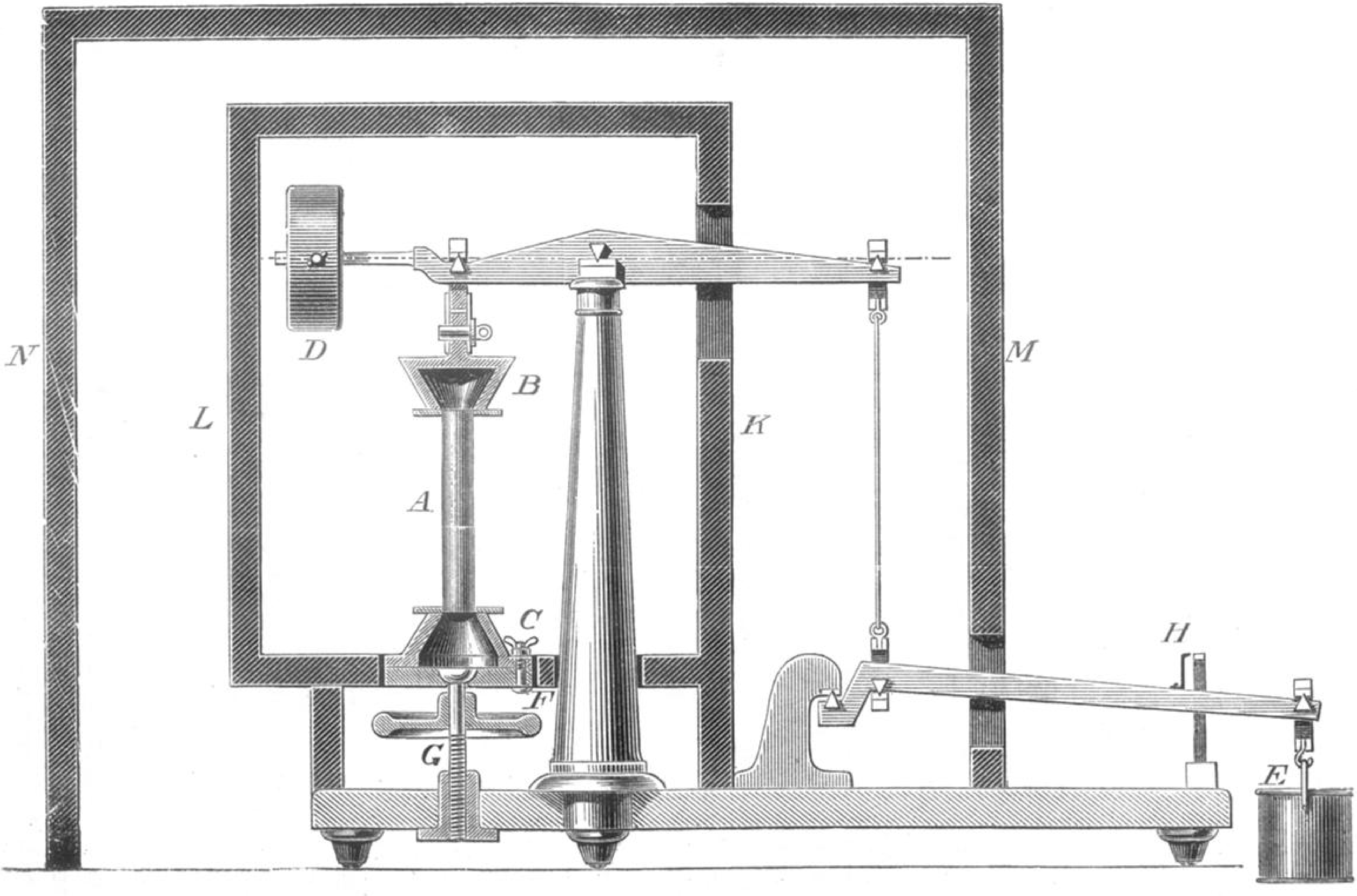


FIG. 3.

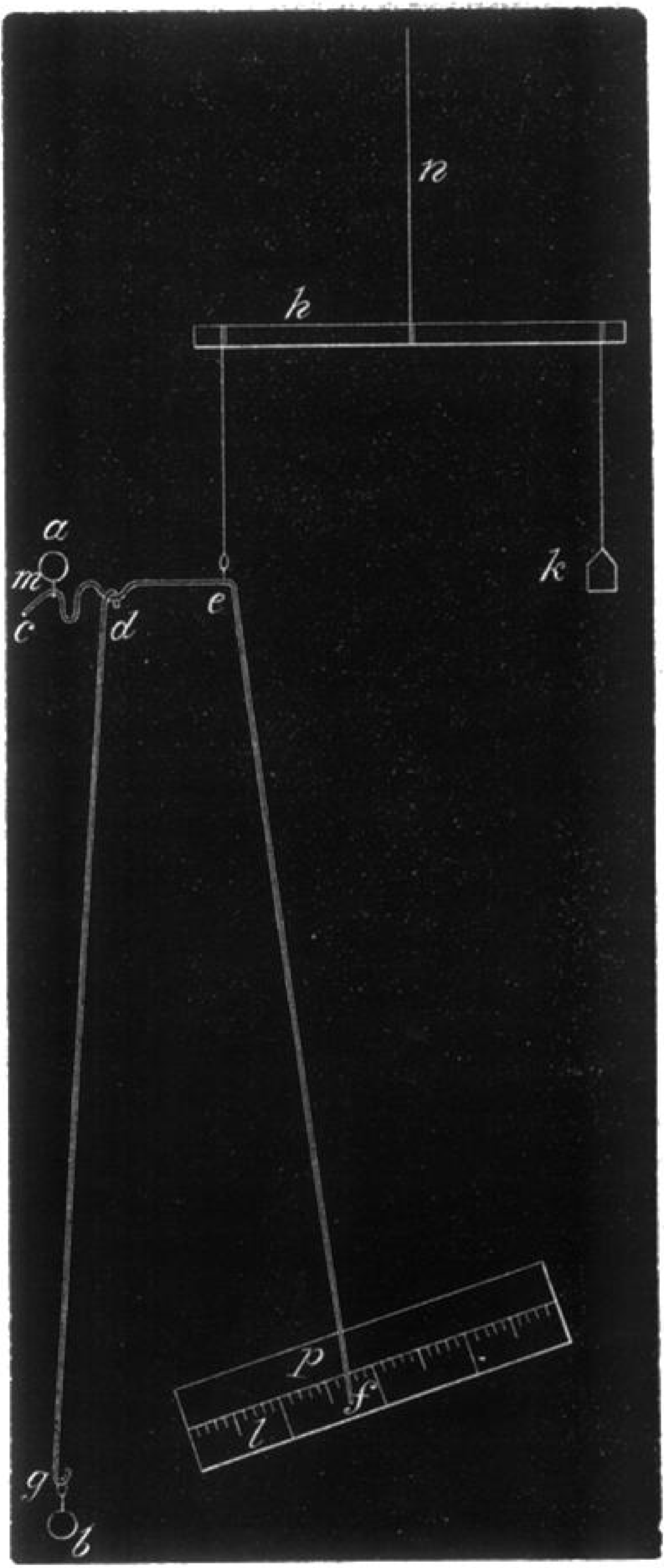


FIG. 4.